MATH 263 Differential Equations and Modelling
Midterm Exam
July 24, 2016 10:00 11:30

Student Name:_____

Instructors: *Prof. Dr. Erol Sezer, Asst. Prof. Dr. Ahmet Yantir*

Question#	1	2	3	4	Total
Question Value	25	25	25	25	100
Your Grade					

[25pt] 1. Find the solution of the initial value problem

$$y' + y^2 \sin t = 0, \quad y(0) = 1.$$

[25pt] 2. Consider the initial value problem

$$\frac{dy}{dx} = \frac{x+4}{y}, \quad y(0) = 4$$

Obtain the approximate values of $y(0.05)$, $y(0.1)$, $y(0.15)$ and $y(0.2)$ using Euler's approximation with step size $h = 0.05$.

[25pt] 3. A 50 kg mass is shot from a cannon straight up with an initial velocity of 10m/s off a bridge that is 100 meters above the ground. If air resistance is given by $5v$,

a) Set up an initial value problem that models the problem.

b) Determine the velocity of the mass when it hits the ground.

[25pt] 4. Use Laplace transform to find the solution of the following IVP:

$$y'' + 2y' + 2y = e^{-t}, \quad y(0) = 2, \quad y'(0) = 1.$$

GOOD LUCK!

MATH 263

Differential Equations and Dynamical Systems

Midterm Exam

October 30, 2017 17:30-18:45

Student Name:_____

Department:_____

Instructors: *Prof. Dr. Erol Sezer, Assist. Prof. Dr. Ahmet Yantir*

Question#	1	2	3	4	Total
Question Value	25	25	25	25	100
Your Grade					

INSTRUCTIONS

1. 75 mins, Open book/notes. No cell phones.
2. Answer each question on its page. Explain your results.
3. Mark your instructor.
4. Observe the honor code. Read and sign the statement below.

   No help is received, given, or observed.

   SIGN:

[25pt] 1. Find the solution of the initial value problem

$$y' = -0.1y^2, \quad y(0) = 10.$$

[25pt] 2. Find the solution of the initial value problem

$$y'' + 10y' + 25y = 25, \quad y(0) = y'(0) = 0$$

and roughly plot the solution.

[25pt] 3. Use Euler's method with with step size $h = 0.1$ to find first five values of numerical solution of the IVP

$$\frac{dy}{dt} = -y - t, \quad y(0) = 1$$

[25pt] 4. A system is described by the initial value problem

$$y' + y = u(t), \quad y(0) = 0.$$

You would like to have a continuous solution $y(t)$ such that $y(t)$=1 for (large t) $t \geq 2$. To achieve this, you apply an input for $u(t)$ in the form:

Find u_1 and u_2.

GOOD LUCK!

MATH 263

Differential Equations and Dynamical Systems

Midterm Exam

23 November 2016 18:30-19:30

Student Name:_____

Instructors: *Prof. Dr. Erol Sezer, Assist. Prof. Dr. Ahmet Yantir*

Question#	1	2	3	4	Total
Question Value	25	25	25	25	100
Your Grade					

INSTRUCTIONS

1. 60 mins, Open book/notes. No cell phones/calculators
2. Answer each question on its page. Explain your results.
3. Observe the honor code. Read and sign the statement below.

   No help is received, given, or observed.

   SIGN:

[25pt] 1. Find the solution of the initial value problem

$$y' = \frac{2t}{y + t^2 y}, \quad y(0) = 0$$

[25pt] 2. Consider the initial value problem

$$\frac{dy}{dt} = \frac{t-2}{y}, \quad y(0) = 2$$

Obtain the approximate values of $y(0.1)$, $y(0.2)$, $y(0.3)$ and $y(0.4)$ using Euler's approximation with step size $h = 0.1$.

[25pt] 3. A $U-$ shaped cup having 75 cm^2 cross-section area on one side and 150 cm^2 cross-section area on the other side is filled with water. If the initial height is 64 cm on the narrow side and 28 cm on the large side and the flow rate is proportional to $\sqrt{h_1 - h_2}$ with $\alpha = 100 cm^{5/2}/s$,

- (a) Relate h_1 and h_2 and/or their derivatives to q.
- (b) Construct a differential equation in q that describes the system.
- (c) Find the steady state values of q, h_1 and h_2.
- (d) Find q as a function of t.

[25pt] 4. Solve the initial value problem by using Laplace transform:

$$y'' + 2y' + 5y = 0, \quad y(0) = 2, \; y'(0) = -1$$

GOOD LUCK!

MATH 263

Differential Equations and Dynamical Systems

Midterm Exam

March 20, 2017 17:30-18:45

Student Name:_____

Department:_____

Instructors: *Assist. Prof. Dr. Ahmet Yantir*

Question#	1	2	3	4	Total
Question Value	25	25	25	25	100
Your Grade					

INSTRUCTIONS

1. 75 mins, Open book/notes. No cell phones.
2. Answer each question on its page. Explain your results.
3. Observe the honor code. Read and sign the statement below.

   No help is received, given, or observed.

   SIGN:

[25pt] 1. Find the general solution of the initial value problem

$$\frac{dx}{dt} = -\frac{te^t}{x}, \quad y(0) = 2.$$

[25pt] 2. Find the general solution of the followinf nonhomogenous differential equation

$$x'' - 2x' - 8x = 4e^{2t} - 24.$$

[25pt] 3. Use improved Euler's method with with step size $h = 0.1$ to find first four values ($x(0.1)$, $x(0.2)$, $x(0.3)$, $x(0.4)$) of numerical solution of the IVP

$$\frac{dx}{dt} - \frac{2}{2t+1}x = 0, \quad x(0) = 1$$

[25pt] 4. A radioactive isotope thorium-234 disintegrates at a rate proportional to the present amount. If 100 mg of this material is reduced to 82 mg in 50 days, find an expression for the amount at any time t. How long does it take for the mass to decay 50 mg from initial amount?

GOOD LUCK!

MATH 2263

Differential Equations and Dynamical Systems

Midterm Exam

November 12, 2017 18:30-19:30

Student Name:_____

Department:_____

Instructors: *Prof. Dr. Erol Sezer, Assist. Prof. Dr. Ahmet Yantir*

Question#	1	2	3	4	Total
Question Value	25	25	25	25	100
Your Grade					

INSTRUCTIONS

1. 60 mins, Open book/notes. No cell phones.
2. Answer each question on its page. Explain your results.
3. Mark your instructor.
4. Observe the honor code. Read and sign the statement below.

   No help is received, given, or observed.

   SIGN:

[25pt] 1. Find the solution of the initial value problem

$$x' = 5 - 0.05x^2, \quad x(0) = 0.$$

[25pt] 2. Find the solution of the initial value problem

$$y'' - 2y' + 10y = 10e^{2t} + 10t - 2, \quad y(0) = 2, \ = y'(0) = 1$$

[25pt] 3. Use Euler's method and improved Euler's method with step size $h = 0.2$ to complete the following table for the IVP

$$\frac{dy}{dt} = -2t, \quad y(0) = 1$$

x	Euler Method	Improved Euler Method	Exact Solution
x_0=0	y_0=1	y_0=1	y_0=1
x_1=0.2	y_1 =	y_1 =	y_1 =
$x_2 = 0.4$	y_2 =	y_2 =	y_2 =

[25pt] 4. Kinetic energy of an object moving in a straight line is known to decrease at a rate proportional to v^2 where v is the speed of the object and $K = \frac{1}{2}mv^2$ is the kinetic energy. If the proportionality constant is $0.1m$ obtain a differential equation that models either v or K.

GOOD LUCK!

MATH 2263

Differential Equations and Dynamical Systems

Midterm Exam

November 12, 2017 18:30-19:30

tudent Name:_____

epartment:_____

nstructors: *Prof. Dr. Erol Sezer, Assist. Prof. Dr. Ahmet Yantir*

Question#	1	2	3	4	Total
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STRUCTIONS

1. 60 mins, Open book/notes. No cell phones.
2. Answer each question on its page. Explain your results.
3. Mark your instructor.
4. Observe the honor code. Read and sign the statement below.

No help is received, given, or observed.

SIGN:

[25pt] 1. Find the solution of the initial value problem

$$x' = 5 - 0.05x^2, \quad x(0) = 0.$$

Solution

$$\frac{dx}{dt} = -0.05\,(x^2 - 100) \Rightarrow \frac{dx}{x^2-100} = -0.05\,dt$$

$$\frac{1}{x^2-100} = \frac{1}{(x-10)(x+10)} = \frac{1/20}{x-10} - \frac{1/20}{x+10}$$

$$\left(\frac{1}{x-10} - \frac{1}{x+10}\right)dx = -20(0.05)\,dt = -dt$$

$$\ln\left|\frac{x-10}{x+10}\right| = -t + c_1 \Rightarrow \frac{x-10}{x+10} = ce^{-t}$$

Initial condition $\Rightarrow c = -1$.

$$\Rightarrow \frac{10-x}{10+x} = e^{-t}$$

$$\Rightarrow x = 10\,\frac{1-e^{-t}}{1+10^{-t}}$$

[25pt] 2. Find the solution of the initial value problem

$$y'' - 2y' + 10y = 10e^{2t} + 10t - 2, \quad y(0) = 2, \; = y'(0) = 1$$

Solution

Characteristic equation : $s^2 - 2s + 10 = 0 \Rightarrow s_{1,2} = 1 \pm 3i$

Complementary solution : $y_c = c_1 e^t \cos 3t + c_2 e^t \sin 3t$ } 5

Assume a particular sol. : $y_p = Ae^{2t} + Bt + C$ ⇒ 5

$y_p' = 2Ae^{2t} + B$

$y_p'' = 4Ae^{2t}$

Substitute (4)

$$(4A - 4A + 10A)e^{2t} + 10Bt + (-2B + 10C) = 10e^{2t} + 10t - 2$$

$\Rightarrow A = B = 1, \; C = 0$ 6

General solution :

$$y = e^t(c_1 \cos 3t + c_2 \sin 3t) + e^{2t} + t$$

$$y' = e^t((c_1 + 3c_2)\cos 3t + (c_2 - 3c_1)\sin 3t) + 2e^{2t} + 1$$

5 (I.C. $y(0) = c_1 + 1 = 2$, $y'(0) = c_1 + 3c_2 + 3 = 1$ } $c_1 = 1$, $c_2 = -1$

Solution : $y = e^t(\cos 3t - \sin 3t) + e^{2t} + t$

[25pt] 3. Use Euler's method and improved Euler's method with step size $h = 0.2$ to complete the following table for the IVP

$$\frac{dy}{dt} = -2t, \quad y(0) = 1$$

x	Euler Method	Improved Euler Method	Exact Solution
x_0=0	y_0=1	y_0=1	y_0=1
x_1=0.2	$y_1 = 1$	$y_1 = 0.96$	$y_1 = 0.96$
$x_2 = 0.4$	$y_2 = 0.92$	$y_2 = 0.84$	$y_2 = 0.84$

Solution:

Euler:

k	0	1	2
t_k	0	0.2	0.4
y_k	1	1.	0.92
$m_k = -2t_k$	0	-0.4	
$y_{k+1} = y_k + 0.2m_k$	1	$1 - 0.08 = 0.92$	

Improved Euler:

k	0	1	2
t_k	0	0.2	0.4
y_k	1	0.96	0.84
$m_{1k} = -2t_k$	0	-0.4	
$m_{2k} = -2(t_k + 0.2)$	-0.4	-0.8	
$m_k = \frac{m_{1k} + m_{2k}}{2}$	-0.2	-0.6	
$y_{k+1} = y_k + 0.2m_k$	$1 - 0.04 = 0.96$	$0.96 - 0.12 = 0.84$	

Exact: $y = 1 - t^2$

$y_1 = 1 - (0.2)^2 = 0.96$

$y_2 = 1 - (0.4)^2 = 0.84$

[25pt] 4. Kinetic energy of an object moving in a straight line is known to decrease at a rate proportional to v^2 where v is the speed of the object, and $K = \frac{1}{2}mv^2$ is the kinetic energy. If the proportionality constant is $0.1m$ obtain a differential equation that models either v or K.

GOOD LUCK!

Solution

$$K = \frac{1}{2}mv^2 \quad \Rightarrow \quad v^2 = \frac{2K}{m}$$

a) $\frac{dK}{dt} = mv\frac{dv}{dt} = -0.1mv^2 \Rightarrow \frac{dv}{dt} = -0.1v$

b) $\frac{dK}{dt} = -0.1mv^2 = -0.2K$

Check for consistency

a) $v = v_0 e^{-0.1t} \Rightarrow K = \frac{1}{2}mv_0^2 e^{-0.2t} = K_0 e^{-0.2t}$

b) $K = K_0 e^{-0.2t}$